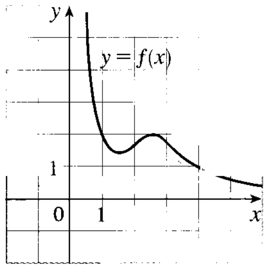
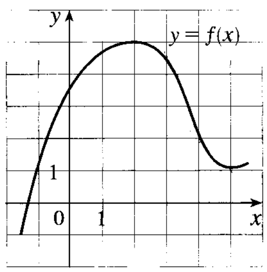
# 2.9 Exercises

**1–3** ▢ Use the given graph to estimate the value of each derivative. Then sketch the graph of $f'$.

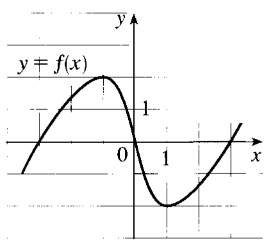
**1.** (a) $f'(1)$
(b) $f'(2)$
(c) $f'(3)$
(d) $f'(4)$




**2.** (a) $f'(0)$
(b) $f'(1)$
(c) $f'(2)$
(d) $f'(3)$
(e) $f'(4)$
(f) $f'(5)$




**3.** (a) $f'(-3)$
(b) $f'(-2)$
(c) $f'(-1)$
(d) $f'(0)$
(e) $f'(1)$
(f) $f'(2)$
(g) $f'(3)$




**4.** Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.

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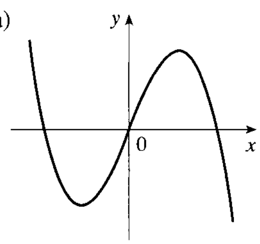
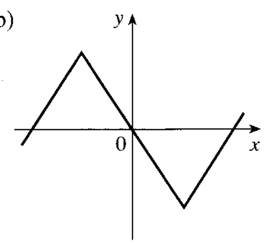
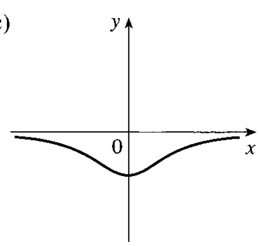
(a)



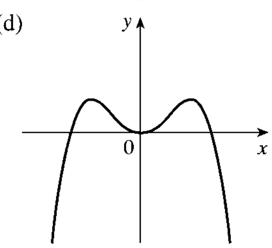

(b)



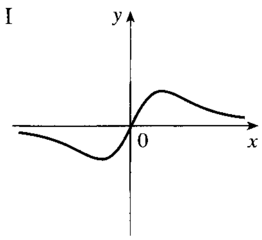

(c)



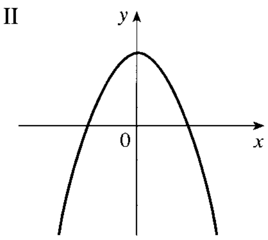

(d)



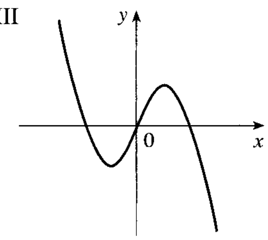

I



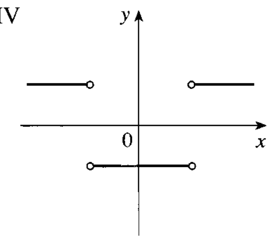

II




III

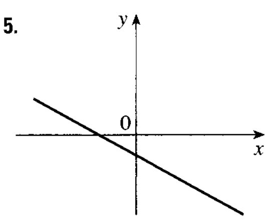



IV

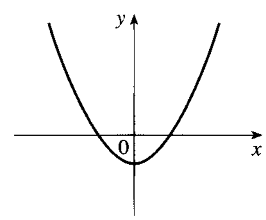



**5–13** ▢ Trace or copy the graph of the given function $f$. (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of $f'$ below it.

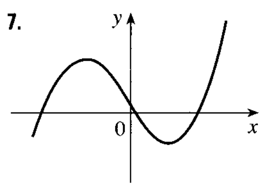
**5.**



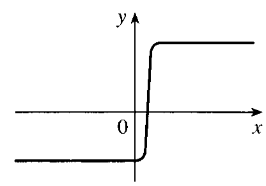

**6.**



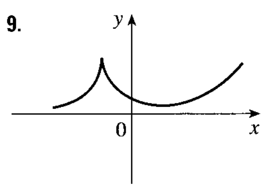

**7.**



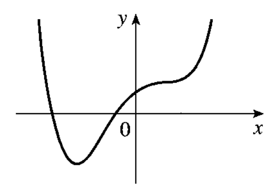

**8.**



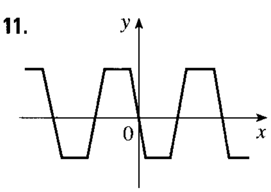

**9.**



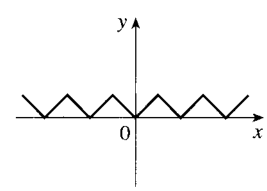

**10.**



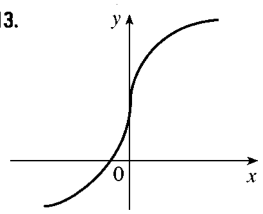

**11.**




**12.**



13.




**14–16** □ Make a careful sketch of the graph of $f$ and below it sketch the graph of $f'$ in the same manner as in Exercises 5–13. Can you guess a formula for $f'(x)$ from its graph?

**14.** $f(x) = \sin x$ **15.** $f(x) = e^x$

**16.** $f(x) = \ln x$

**17.** Let $f(x) = x^2$.
(a) Estimate the values of $f'(0)$, $f'(\frac{1}{2})$, $f'(1)$, and $f'(2)$ by using a graphing device to zoom in on the graph of $f$.
(b) Use symmetry to deduce the values of $f'(-\frac{1}{2})$, $f'(-1)$, and $f'(-2)$.
(c) Use the results from parts (a) and (b) to guess a formula for $f'(x)$.
(d) Use the definition of a derivative to prove that your guess in part (c) is correct.

**18.** Let $f(x) = x^3$.
(a) Estimate the values of $f'(0)$, $f'(\frac{1}{2})$, $f'(1)$, $f'(2)$, and $f'(3)$ by using a graphing device to zoom in on the graph of $f$.
(b) Use symmetry to deduce the values of $f'(-\frac{1}{2})$, $f'(-1)$, $f'(-2)$, and $f'(-3)$.
(c) Use the values from parts (a) and (b) to graph $f'$.
(d) Guess a formula for $f'(x)$.
(e) Use the definition of a derivative to prove that your guess in part (d) is correct.

**19–27** □ Find the derivative of the given function using the definition of derivative. State the domain of the function and the domain of its derivative.

**19.** $f(x) = 5x + 3$ **20.** $f(x) = 5 - 4x + 3x^2$

**21.** $f(x) = x^3 - x^2 + 2x$ **22.** $f(x) = x + \sqrt{x}$

**23.** $g(x) = \sqrt{1 + 2x}$ **24.** $f(x) = \dfrac{x + 1}{x - 1}$

**25.** $G(x) = \dfrac{4 - 3x}{2 + x}$ **26.** $g(x) = \dfrac{1}{x^2}$

**27.** $f(x) = x^4$

**28.** (a) Sketch the graph of $f(x) = \sqrt{6 - x}$ by starting with the graph of $y = \sqrt{x}$ and using the transformations of Section 1.3.
(b) Use the graph from part (a) to sketch the graph of $f'$.
(c) Use the definition of a derivative to find $f'(x)$. What are the domains of $f$ and $f'$?
(d) Use a graphing device to graph $f'$ and compare with your sketch in part (b).

**29.** (a) If $f(x) = x - (2/x)$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of $f$ and $f'$.

**30.** (a) If $f(t) = 6/(1 + t^2)$, find $f'(t)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of $f$ and $f'$.

**31.** The unemployment rate $U(t)$ varies with time. The table (from the Bureau of Labor Statistics) gives the percentage of unemployed in the U. S. labor force from 1988 to 1997.

| $t$ | $U(t)$ | $t$ | $U(t)$ |
|---|---|---|---|
| 1988 | 5.5 | 1993 | 6.9 |
| 1989 | 5.3 | 1994 | 6.1 |
| 1990 | 5.6 | 1995 | 5.6 |
| 1991 | 6.8 | 1996 | 5.4 |
| 1992 | 7.5 | 1997 | 4.9 |

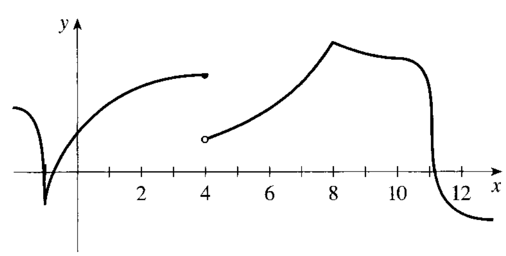
(a) What is the meaning of $U'(t)$? What are its units?
(b) Construct a table of values for $U'(t)$.

**32.** Let the smoking rate among high-school seniors at time $t$ be $S(t)$. The table (from the Institute of Social Research, University of Michigan) gives the percentage of seniors who reported that they had smoked one or more cigarettes per day during the past 30 days.

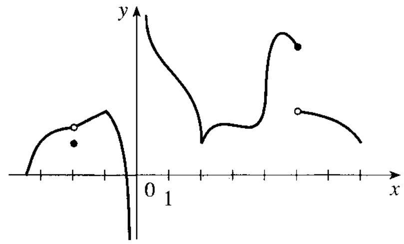
| $t$ | $S(t)$ | $t$ | $S(t)$ |
|---|---|---|---|
| 1978 | 27.5 | 1988 | 18.1 |
| 1980 | 21.4 | 1990 | 19.1 |
| 1982 | 21.0 | 1992 | 17.2 |
| 1984 | 18.7 | 1994 | 19.4 |
| 1986 | 18.7 | 1996 | 22.2 |

(a) What is the meaning of $S'(t)$? What are its units?
(b) Construct a table of values for $S'(t)$.
(c) Graph $S$ and $S'$.
(d) How would it be possible to get more accurate values for $S'(t)$?

**33.** The graph of $f$ is given. State, with reasons, the numbers at which $f$ is not differentiable.




**34.** The graph of $g$ is given.
(a) At what numbers is $g$ discontinuous? Why?
(b) At what numbers is $g$ not differentiable? Why?




**35.** Graph the function $f(x) = x + \sqrt{|x|}$. Zoom in repeatedly, first toward the point $(-1, 0)$ and then toward the origin. What is different about the behavior of $f$ in the vicinity of these two points? What do you conclude about the differentiability of $f$?

**36.** Zoom in toward the points $(1, 0)$, $(0, 1)$, and $(-1, 0)$ on the graph of the function $g(x) = (x^2 - 1)^{2/3}$. What do you notice? Account for what you see in terms of the differentiability of $g$.

**37.** Let $f(x) = \sqrt[3]{x}$.
(a) If $a \neq 0$, use Equation 2.8.3 to find $f'(a)$.
(b) Show that $f'(0)$ does not exist.
(c) Show that $y = \sqrt[3]{x}$ has a vertical tangent line at $(0, 0)$. (Recall the shape of the graph of $f$. See Figure 13 in Section 1.2.)

**38.** (a) If $g(x) = x^{2/3}$, show that $g'(0)$ does not exist.
(b) If $a \neq 0$, find $g'(a)$.
(c) Show that $y = x^{2/3}$ has a vertical tangent line at $(0, 0)$.
(d) Illustrate part (c) by graphing $y = x^{2/3}$.

**39.** Show that the function $f(x) = |x - 6|$ is not differentiable at 6. Find a formula for $f'$ and sketch its graph.

**40.** Where is the greatest integer function $f(x) = [\![x]\!]$ not differentiable? Find a formula for $f'$ and sketch its graph.

**41.** (a) Sketch the graph of the function $f(x) = x|x|$.
(b) For what values of $x$ is $f$ differentiable?
(c) Find a formula for $f'$.

**42.** The **left-hand** and **right-hand derivatives** of $f$ at $a$ are defined by

$$f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$

and

$$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

if these limits exist. Then $f'(a)$ exists if and only if these one-sided derivatives exist and are equal.
(a) Find $f'_-(4)$ and $f'_+(4)$ for the function

$$f(x) = \begin{cases} 0 & \text{if } x \leqslant 0 \\ 5 - x & \text{if } 0 < x < 4 \\ \dfrac{1}{5 - x} & \text{if } x \geqslant 4 \end{cases}$$

(b) Sketch the graph of $f$.
(c) Where is $f$ discontinuous?
(d) Where is $f$ not differentiable?

**43.** Recall that a function $f$ is called *even* if $f(-x) = f(x)$ for all $x$ in its domain and *odd* if $f(-x) = -f(x)$ for all such $x$. Prove each of the following.
(a) The derivative of an even function is an odd function.
(b) The derivative of an odd function is an even function.

**44.** When you turn on a hot-water faucet, the temperature $T$ of the water depends on how long the water has been running.
(a) Sketch a possible graph of $T$ as a function of the time $t$ that has elapsed since the faucet was turned on.
(b) Describe how the rate of change of $T$ with respect to $t$ varies as $t$ increases.
(c) Sketch a graph of the derivative of $T$.

**45.** Let $\ell$ be the tangent line to the parabola $y = x^2$ at the point $(1, 1)$. The *angle of inclination* of $\ell$ is the angle $\phi$ that $\ell$ makes with the positive direction of the $x$-axis. Calculate $\phi$ correct to the nearest degree.